

Brightness temperature — obtaining physical properties of non-equipartition plasma.

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ABSTRACT

The limit on the intrinsic brightness temperature, attributed to ‘Compton catastrophe’, has been established being 10^{12} K. Somewhat lower limit of the order of $10^{11.5}$ K is implied if we assume that the radiating plasma is in equipartition with the magnetic field — the idea that explained why the observed cores of active galactic nuclei sustained the limit lower than the ‘Compton catastrophe’. Recent observations with unprecedented high resolution by the RADIOASTRON revealed systematic exceed in the observed brightness temperature. We propose means of estimating the degree of non-equipartition regime in AGN cores. Coupled with the core-shift measurements the method allows us to estimate independently the magnetic field strength and the particles number density at the core. We show that the ratio of magnetic energy to radiating plasma energy is of the order of 10^{-5} , which means the flow in the core is dominated by the particle energy. We show that the magnetic field obtained by the brightness temperature measurements may be underestimated. We propose for the relativistic jets with small viewing angles the non-uniform MHD model, and obtain the expression for the magnetic field amplitude about two orders higher than that for the uniform model. These magnetic field amplitudes are consistent with the limiting magnetic field suggested by the ‘magnetically arrested disk’ model.

Key words: galaxies: active — galaxies: jets — quasars: general — radio continuum: galaxies — radiation mechanisms: non-thermal

1 INTRODUCTION

The previous observations of active galactic nuclei (AGN) in radio band all have limited the cores brightness temperature by 10^{12} K. This phenomenon has been explained in Kellermann & Pauliny-Toth (1969) as being an outcome of so called “inverse Compton catastrophe”.

It can be illustrated by such an argument (Kirk, Melrose & Priest 1994). Suppose we have an electron moving in magnetic field B with the velocity v , $\beta = v/c$, c is a speed of light, and with corresponding Lorentz factor γ . It radiates synchrotron radiation with the power (see, e.g., Rybicki & Lightman (1979))

$$P_S = \frac{4}{3} \sigma_T c \beta^2 \gamma^2 U_B. \quad (1)$$

Here $\sigma_T = 8\pi/3r_0^2$ is Thomson cross-section, $r_0 = e^2/mc^2$ is an electron classical radius, e and m are electron charge and mass, c is a speed of light, and $U_B = B^2/8\pi$ is the magnetic energy density. The same electron loses its energy

undergoing the inverse Compton scattering of photons, the power being

$$P_C = \frac{4}{3} \sigma_T c \beta^2 \gamma^2 U_{ph}, \quad (2)$$

where the photon energy density

$$U_{ph} = \int \epsilon dn(\epsilon), \quad (3)$$

with photon energy distribution $n(\epsilon)$. The photon energy density U_{ph} comprises of synchrotron photons U_{ph0} , once Comptonized photons U_{ph1} , and so forth. The full power of Compton losses is described by Kirk, Melrose & Priest (1994)

$$U_{ph} = U_{ph0} \left[1 + \frac{U_{ph0}}{U_B} + \left(\frac{U_{ph0}}{U_B} \right)^2 + \dots \right] = \frac{U_{ph0}}{1 - U_{ph0}/U_B}. \quad (4)$$

If $U_{ph0} = U_B$, the power P_C diverges, which is referred to by the “inverse Compton catastrophe”.

The result, obtained by Kellermann & Pauliny-Toth (1969) is the limiting brightness temperature 10^{12} K, beyond which the “inverse Compton catastrophe” takes place.

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The question was how does the source “know” this limit and sustains its brightness temperature below the limit. The answer has been proposed by Readhead (1994). There is another limit on the brightness temperature — the so called equipartition temperature. If radiating plasma and magnetic field are in energy equipartition in the source, the corresponding temperature T_{eq} is just below the limiting by the inverse Compton catastrophe one.

However, recent observations of AGN radio cores with the high-resolution *RADIOASTRON* program Kovalev et al. (2016) questioned the existence of such a limit, since there are observations that systematically show the brightness temperatures greater than not only T_{eq} , but also 10^{12} K.

In this work we do not approach a question what is the physical process underlying such extreme brightness temperatures. We address the question of obtaining the non-equipartition physical parameters of the radiating domain such as the magnetic field B , particle number density n and the measure of non-equipartition Σ . This is an important issue. Indeed, the analytical and numerical modeling (Beskin & Nokhrina 2006; Komissarov et al. 2007; Lyubarsky 2009; Tchekhovskoy et al. 2008, 2009) support the idea, that relativistic jets from AGN must be in equipartition regime. In particular, the magnetization parameter

$$\sigma = \frac{B^2}{4\pi n m c^2 \Gamma^2}, \quad (5)$$

which is the ratio of Poynting vector flux to plasma kinetic energy flux, must be unity. Here Γ is the bulk Lorentz factor of a jet, and n is a proper particle number density. However, it is believed that only the small portion of particles radiate. Indeed, the bulk plasma velocity, dictated by the magneto-hydrodynamics, is exactly the drift velocity in crossed electric and magnetic fields. So, a cold plasma, moving with the drift velocity, does not radiate. To produce the radiation, cold plasma must be disturbed and accelerated, the power law

$$dn = k_e \gamma^{-p} d\gamma, \quad \gamma \in [\gamma_{\min}; \gamma_{\max}], \quad (6)$$

being used to describe the energy distribution of the radiating plasma. Here k_e is an amplitude of the electron energy distribution. To model the radiation of particles in the magnetic field, we employ synchrotron emission and self-absorption. Indeed, spectral energy distributions (SED) performed for blazars (see e.g. Abdo (2011)) demonstrate that at the low energy the Compton part of the radiation does not play an important role.

As to acceleration process itself, there are two main processes which may account for it. One is the particle acceleration on shocks. This process can hardly account for the observed radiation, since it has been shown (see, e.g., Kirk, Melrose & Priest (1994)), that the acceleration is not efficient for the magnetized shocks. As the possibility for a particle once accelerated to return to the shock front is suppressed, the Fermi acceleration mechanism does not work. The second process is the reconnection of magnetic field. It accelerates about one per cent of particles very effectively (Sironi et al. 2013), producing a power-law spectrum, with maximum particles energy growing with the time of numerical simulation.

2 JET PARAMETERS IN BLANDFORD–KÖNIGL MODEL

The following model is used to explain the properties of compact bright features observed in radio band (see Gould (1979); Lind & Blandford (1985); Lobanov (1998); Zdziarski et al. (2015)): the radiation domain is either a uniform “plasmoid” or a uniform excited part of a continuous jet. The position of this radiating spherical (Gould 1979) domain along the jet r defines the amplitudes of the particle number density and of the magnetic field (with uniform distribution across the radiating domain) according to Blandford–Königl model (Blandford & Königl 1979) as

$$B(r) = B_0 \left(\frac{r_0}{r} \right), \quad k_e(r) = k_{e,0} \left(\frac{r_0}{r} \right)^2. \quad (7)$$

Here B_0 and $k_{e,0}$ are magnetic field and particle number density amplitude at a distance r_0 .

The model has been used to obtain such physical parameters of jets as a magnetic field, a particle number density (Lobanov 1998; Hirotani 2005; O’Sullivan & Gabuzda 2009), and multiplicity parameter and Michel’s (Michel 1969) magnetization parameter (Nokhrina et al. 2015) using a core-shift effect — the observed shift of position of radio cores at different frequencies. All these result are based on the equipartition assumption in different formulation — either the energy densities of magnetic field and radiating particles are equal, or the fluxes of Poynting vector and total particle kinetic energy are equal with the number of radiating particles consisting about 1 per cent of the total particle number density.

2.1 Magnetic field

The observed flux, or observed brightness temperature, can be used to estimate the magnetic field in the radiating domain (Zdziarski et al. 2015). The observed spectral flux S_ν of a core at the frequency ν can be expressed on the one hand through the brightness temperature T_b as

$$S_\nu = \frac{2\pi\nu^2\theta^2}{c^2} k_B T_b, \quad (8)$$

where θ is the angular size of a radiating domain. On the other hand the flux for the optically thick uniform source of radius R at the distance d can be written using the spectral photon emission rate ρ_ν and effective absorption coefficient \mathfrak{a}_ν as (Gould 1979):

$$S_\nu = \pi \hbar \nu \frac{\rho_\nu}{\mathfrak{a}_\nu} \frac{R^2}{d^2} u(2R\mathfrak{a}_\nu), \quad (9)$$

and the function of the optical depth $u(2R\mathfrak{a}_\nu)$ is defined in Gould (1979). The emission and absorption coefficients for the synchrotron-self-Compton model can be written in a jet frame (primed), i.e. in a frame where the electric field vanishes, as (Ginzburg & Syrovatskii 1964; Blumenthal & Gould 1970; Gould 1979):

$$\rho'_{\nu'} = 4\pi \left(\frac{3}{2} \right)^{(p-1)/2} a(p) \alpha k'_e \left(\frac{\nu'_{B'}}{\nu'} \right)^{(p+1)/2}, \quad (10)$$

$$\mathfrak{a}'_{\nu'} = c(p) r_0^2 k'_e \left(\frac{\nu_0}{\nu'} \right) \left(\frac{\nu'_{B'}}{\nu'} \right)^{(p+2)/2}. \quad (11)$$

Here $\nu'_{B'} = eB'/mc$ is a gyrofrequency in the fluid frame,

\hbar is the Planck constant, $\alpha = e^2/\hbar c$ is the fine structure constant, and the functions $a(p)$ and $c(p)$ of the electron distribution spectral index p are defined in (Gould 1979).

Equations (8) and (9) are written in an observer frame. However, the spectral flux is calculated in a jet (primed) frame using (10) and (11), where it is expressed as a function of a frequency ν' , magnetic field B' and particle number density amplitude k'_e in the jet frame. In order to rewrite a flux and a brightness temperature into observer frame we use the Lorentz invariant (Rybicki & Lightman 1979) S_ν/ν^3 . To express a magnetic field and a particle number density in the nucleus frame, and a frequency in an observer frame, we use the following relations. A particle number density amplitude k'_e in a fluid frame correlates with its value k_e in the nucleus frame as

$$k'_e = k_e/\Gamma, \quad (12)$$

an observed frequency transforms from the fluid frame into observer frame as

$$\nu' = \nu_{\text{obs}} \frac{1+z}{\delta}, \quad (13)$$

and a brightness temperature $T_{b,\text{obs}} = T_b\delta/(1+z)$. Here z is a cosmological red-shift of a source and a Doppler factor of a flow $\delta = [\Gamma(1 - \beta \cos \varphi)]^{-1}$. The viewing angle of a jet is φ . We assume that the toroidal component of a magnetic field dominates the jet radiating region outside the light cylinder with its position defined by $R_L = c/\Omega_F$. Indeed, the MHD analytical (Beskin 1997; Narayan et al. 2007; Tchekhovskoy et al. 2008; Lyubarsky 2009; Nokhrina et al. 2015) and numerical (Tchekhovskoy & Bromberg 2016) models provide that $B_\varphi \approx B_{\text{pr}}/R_L$. Thus, the magnetic fields transforms from the fluid frame into the nucleus frame as $B \approx B'\Gamma$.

Equating the right-hand sides of equations (8) and (9), we obtain for the magnetic field

$$B = k_0(p) \frac{m^3 c^5}{e} \frac{\Gamma \delta}{1+z} \nu_{\text{obs}} (k_b T_{b,\text{obs}})^{-2}, \quad (14)$$

where the numerical factor k_0 depends on the electrons spectral index and is equal to

$$k_0(p) = 3.6 \cdot 10^{-1}, \quad p = 2. \quad (15)$$

Particular spectral index p may be found by fitting the jet spectrum for a particular source. From the theoretical point of view, it depends on the non-thermal mechanism of particle acceleration in AGN, which is under debate. The first-order Fermi mechanism working at shocks provides $p = 2$ (Blandford & Ostriker 1978). However, the numerical simulations demonstrate that this mechanism works effectively for low magnetization flows (Sironi & Spitkovsky 2011), with $p \approx 2.5$. On the other hand, magnetic reconnection provides means for particle acceleration and formation of power-law spectrum with the spectral index p depending on the flow magnetization (Sironi & Spitkovsky 2014) and ranging from 1.5 to 4. Having in mind the uncertainty of the spectral index, we choose the value $p = 2$ as a fiducial parameter characterizing the non-thermal spectrum of radiating particles. Substituting $p = 2$ we obtain

$$\left(\frac{B}{G}\right) = 7.4 \cdot 10^{-4} \frac{\Gamma \delta}{1+z} \left(\frac{\nu_{\text{obs}}}{\text{GHz}}\right) \left(\frac{T_{b,\text{obs}}}{10^{12}\text{K}}\right)^{-2}. \quad (16)$$

The radio core is observed at the peak spectral flux, with

$\nu_{\text{obs}} = \nu_{\text{peak}}$ for the magnetic field B and radiating particle number density n at the surface of the optical depth equal to unity. For each frequency the position of this surface is different (core-shift effect, see e.g. (Lobanov 1998)), so the defined by equation (16) magnetic field is for the particular position r_{core} of the observed core. The equation (16) does not give us a full information about the magnetic field amplitude, since we do not know the position of the core at the observed frequency. In order to obtain the core position, we need the measurements of the core-shift effect (Lobanov 1998; Sokolovsky et al. 2011; Pushkarev et al. 2012) as well.

2.2 Measure of equipartition

The core-shift effect is a change in the observed position of a core at different frequencies (Lobanov 1998; O'Sullivan & Gabuzda 2009). It is connected with the self-absorption of the synchrotron sources (see e.g. Gould (1979)): due to absorption we observe the surface of the optical thickness equal to unity. Both synchrotron emission rate and the absorption depend on the emitting particle number density and magnetic field magnitudes and distributions. The 'stardart' core-shift formula by Lobanov (1998) has been obtained under certain assumptions: the Blandford-Konigl field and particle number density dependence on r (7) and the equipartition between the radiating plasma and magnetic field. The last assumption has been essential for the results, since measurements of the core-shift allows only to estimate the dependence of magnetic field magnitude on particle number density. The same equipartition assumption has been used to establish the 'equipartition brightness temperature' by Readhead (1994). However, recent observations of the brightness temperature at high resolution provided by RADIOASTRON exceed this 'equipartition' limit, so, as has been indicated by Gómez et al. (2016), there is, probably, no equipartition in a jet. However, the measurements of both brightness temperature and core-shift provides us with the instrument to estimate the magnetic field and particle number density independently (Zdziarski et al. 2015), and thus obtain the measure of 'non-equipartition'.

Let us introduce the radiation magnetization Σ — the ratio of Poynting flux to radiating particle energy flux. Each particle internal energy is given by $mc^2\gamma'$, where γ' is the Lorentz factor of radiating particles with respect to plasma bulk motion. The radiating particle number density n'_{rad} is given in jet bulk motion proper frame. The amplitude k'_e is defined by the radiating particles number density n'_{rad} depending on the magnitude of exponent p as $n'_{\text{rad}} = k'_e f(p)$ with

$$f(p) = \begin{cases} \frac{1}{1-p} (\gamma_{\text{max}}^{1-p} - \gamma_{\text{min}}^{1-p}), & p \neq 1, \\ \ln \frac{\gamma_{\text{max}}}{\gamma_{\text{min}}}, & p = 1. \end{cases} \quad (17)$$

We assume $p \in (1; 2]$, and the Lorentz factor of plasma in nucleus frame is defined by $\gamma = \gamma'\Gamma$. In this case, the magnetization of radiating particles is

$$\Sigma = \frac{\Gamma(2-p)B^2 f(p)}{4\pi mc^2 n_{\text{rad}} (\gamma_{\text{max}}^{2-p} - \gamma_{\text{min}}^{2-p})} \quad (18)$$

for $p \neq 2$, and

$$\Sigma = \frac{\Gamma B^2 f(p)}{4\pi m c^2 n_{\text{rad}} \ln \frac{\gamma_{\text{max}}}{\gamma_{\text{min}}}} \quad (19)$$

for $p = 2$. Here n_{rad} is given in nucleus frame. For the equipartition between magnetic field and radiating particles $\Sigma = 1$. We will be interested in estimates for Σ from the observations. This will allow us to connect the radiation magnetization Σ with the physical properties of the radiating plasma. We will use the non-dimensional function $F_{\Sigma}(p)$ such as

$$\Sigma = \frac{\Gamma B^2}{m c^2 n_{\text{rad}}} F_{\Sigma}(p). \quad (20)$$

The expression connecting the jet physical parameters B and r_{rad} with the position of radiating region r and the observed frequency ν_{obs} has been obtained by Lobanov (1998); Hirotsu (2005); Nokhrina et al. (2015):

$$B^{2+p} n_{\text{rad}}^2 = \nu_{\text{obs}}^{4+p} F_1^{-1} F_2^{-1} r^{-2}, \quad (21)$$

where coefficients

$$F_1 = \frac{c^2(p)(p-1)^2}{5(4+p)} \frac{e^4}{m^2 c^2} \left(\frac{e}{2\pi m c} \right)^{2+p}, \quad (22)$$

and

$$F_2 = \left(\frac{\delta}{\Gamma(1+z)} \right)^{4+p} \left(\frac{2\chi}{\delta \sin \varphi} \right)^2. \quad (23)$$

Here χ is a jet half-opening angle for the conical jet. Using (20), we rewrite n_{rad} as a function of Σ and B , and substituting (14) into (21) we obtain the expression for the flow magnetization in a radiating domain as a function of its position r from the central source, the observed brightness temperature, an observed frequency, and geometrical and velocity factors:

$$\Sigma = 4.1 \cdot 10^3 (1.7 \cdot 10^2)^{-p} C_{\Sigma}(p) \frac{2\chi \Gamma^2}{\delta \sin \varphi} \left(\frac{\delta}{1+z} \right)^{p+5} \times \left(\frac{r}{\text{pc}} \right) \left(\frac{\nu_{\text{obs}}}{\text{GHz}} \right) \left(\frac{T_{\text{b, obs}}}{10^{12} \text{K}} \right)^{-(p+6)}. \quad (24)$$

Here

$$C_{\Sigma}(p) = \frac{F_{\Sigma}(p)}{f(p)} \frac{c(p)}{\sqrt{5(4+p)}} (2\pi)^2 \times \left[2.8(1.5)^{(p-1)/2} \frac{a(p)}{c(p)} \right]^{p+6}. \quad (25)$$

For $p = 2$ we obtain

$$\Sigma = 1.58 \cdot 10^{-5} \frac{2\chi \Gamma^2 \delta^6}{\sin \varphi (1+z)^7} \frac{F_{\Sigma}(2)}{f(2)} \times \left(\frac{r}{\text{pc}} \right) \left(\frac{\nu_{\text{obs}}}{\text{GHz}} \right) \left(\frac{T_{\text{b, obs}}}{10^{12} \text{K}} \right)^{-8}. \quad (26)$$

The expression above has been obtained assuming (i) the radiating region has uniform distribution of n_{rad} and B ; (ii) the radiating domain is optically thick; (iii) the jet is conical with half-opening angle χ , so that the jet geometrical thickness along the line of sight depends on r , χ and φ (see Hirotsu (2005) for details); (iv) we observe the surface of the optical depth approximately equal to unity at the observed frequency ν_{obs} . This allows us to estimate the order of Σ , assuming that r is of the order of a parsec.

However, if we additionally adopt the Blandford–Königl scalings for the magnetic field B and particle number density n_{rad} (7), we will be able to correlate the position of a radiating domain r with the observed frequency ν_{obs} . Indeed, substituting (7) into (21) one obtains the classical expression $\nu_{\text{obs}} r$ proportional to the physical parameters of a jet. The last conclusion is supported by multifrequency observations by Sokolovsky et al. (2011). Thus, if we have, in addition to the measurement of the brightness temperature, the core-shift measurement, we can use it to obtain the radiating domain position. As

$$r \sin \varphi = \theta_d \frac{D_L}{(1+z)^2}, \quad (27)$$

where D_L is a luminosity distance, we introduce

$$\Delta \theta_d = \Phi \left(\frac{1}{\nu_1} - \frac{1}{\nu_2} \right). \quad (28)$$

With $\Delta \theta_d$ being measured for the two frequencies ν_1 and ν_2 , we can calculate Φ in mas GHz and find the observed position of the core at given frequency as

$$r_{\text{core}} = \frac{\Phi D_L}{\nu_{\text{obs}} \sin \varphi (1+z)^2}. \quad (29)$$

Knowing the core-shift we can estimate the radial distance of the observed radiating domain of a jet:

$$\frac{r_{\text{obs}}}{\text{pc}} = \frac{4.8}{\sin \varphi (1+z)^2} \left(\frac{\nu_{\text{obs}}}{\text{GHz}} \right)^{-1} \left(\frac{\Phi}{\text{mas} \cdot \text{GHz}} \right) \left(\frac{D_L}{\text{Gpc}} \right), \quad (30)$$

and, consequently, the magnetization in the observed core as

$$\Sigma = 2.1 \cdot 10^4 (1.7 \cdot 10^2)^{-p} C_{\Sigma}(p) \frac{2\chi \Gamma^2 \delta^{p+4}}{\sin^2 \varphi (1+z)^{p+7}} \times \left(\frac{D_L}{\text{Gpc}} \right) \left(\frac{\Phi}{\text{mas} \cdot \text{GHz}} \right) \left(\frac{T_{\text{b, obs}}}{10^{12} \text{K}} \right)^{-(p+6)}. \quad (31)$$

For $p = 2$ the expression is

$$\Sigma = 7.7 \cdot 10^{-5} \frac{2\chi \Gamma^2 \delta^6}{\sin^2 \varphi (1+z)^9} \frac{F_{\Sigma}(2)}{f(2)} \times \left(\frac{D_L}{\text{Gpc}} \right) \left(\frac{\Phi}{\text{mas} \cdot \text{GHz}} \right) \left(\frac{T_{\text{b, obs}}}{10^{12} \text{K}} \right)^{-8}. \quad (32)$$

2.3 Radiating particles number density

In order to obtain the radiating particle number density in a radiating domain, we substitute (14) into (21):

$$\left(\frac{n_{\text{rad}}}{\text{cm}^{-3}} \right) = 1.1 \cdot 10^{-3} (1.7 \cdot 10^2)^p C_n(p) \times \frac{\Gamma \sin \varphi (1+z)^{p+3}}{2\chi \delta^{p+2}} \left(\frac{r}{\text{pc}} \right)^{-1} \left(\frac{\nu_{\text{obs}}}{\text{GHz}} \right) \left(\frac{T_{\text{b, obs}}}{10^{12} \text{K}} \right)^{p+2}. \quad (33)$$

Here

$$C_n(p) = f(p) \frac{\sqrt{5(p+4)}}{c(p)} \left[2.8(1.5)^{(p-1)/2} \frac{a(p)}{c(p)} \right]^{-(p+2)}. \quad (34)$$

For $p = 2$ we obtain the estimate for radiating particles number density at the region with the position r :

$$\left(\frac{n_{\text{rad}}}{\text{cm}^{-3}}\right) = 4 \cdot 10^4 \frac{\Gamma \sin \varphi (1+z)^5}{2\chi \delta^4} f(2) \times \left(\frac{r}{\text{pc}}\right)^{-1} \left(\frac{\nu_{\text{obs}}}{\text{GHz}}\right) \left(\frac{T_{\text{b, obs}}}{10^{12} \text{K}}\right)^4. \quad (35)$$

Using (30), one can obtain the expression for n_{rad} as a function of the observables

$$\left(\frac{n_{\text{rad}}}{\text{cm}^{-3}}\right) = 2.3 \cdot 10^{-4} (1.7 \cdot 10^2)^p C_n(p) \times \frac{\Gamma \sin^2 \varphi (1+z)^{p+5}}{2\chi \delta^{p+2}} \left(\frac{D_L}{\text{Gpc}}\right)^{-1} \left(\frac{\Phi}{\text{mas} \cdot \text{GHz}}\right)^{-1} \times \left(\frac{\nu_{\text{obs}}}{\text{GHz}}\right)^2 \left(\frac{T_{\text{b, obs}}}{10^{12} \text{K}}\right)^{p+2}. \quad (36)$$

For $p = 2$

$$\left(\frac{n_{\text{rad}}}{\text{cm}^{-3}}\right) = 8.2 \cdot 10^3 \frac{\Gamma \sin^2 \varphi (1+z)^7}{2\chi \delta^4} f(2) \times \left(\frac{D_L}{\text{Gpc}}\right)^{-1} \left(\frac{\Phi}{\text{mas} \cdot \text{GHz}}\right)^{-1} \left(\frac{\nu_{\text{obs}}}{\text{GHz}}\right)^2 \left(\frac{T_{\text{b, obs}}}{10^{12} \text{K}}\right)^4. \quad (37)$$

2.4 Physical parameters in the sources with extreme brightness temperatures

The above estimates we can apply to two objects with measured brightness temperature and core-shift. The equations (16), (32) and (37) permit us to obtain estimates for the radiating particles magnetization Σ , magnetic field B and radiating particles number density n_{rad} in the observed radio core (radiating domain) if we have precise enough measurement of the brightness temperature. On the other hand, if we have the lower limit for the brightness temperature (Lobanov 2015), these expressions provide the lower limit for particle number density n_{rad} and upper limits for the magnetic field B and magnetization parameter Σ .

We will calculate the magnetic field B , particle number density (in nucleus frame) n_{rad} and magnetization (measure of equipartition) Σ for the blazars BL Lac and 3C273 basing on the measurements of the core brightness temperature by Gómez et al. (2016) and Kovalev et al. (2016). The other parameters we need are the Doppler factor, Lorentz factor of a flow, the observation angle φ , red shift z , and the half-opening angle χ . The red shift and apparent velocity

$$\beta_{\text{app}} = \frac{\beta \sin \varphi}{1 - \beta \cos \varphi} \quad (38)$$

we take from Lister et al. (2013).

There are several approaches to deduction of a Doppler factor δ from the observed jet parameters. The first one employs the relation $\varphi \approx \gamma^{-1}$ and provides $\delta_{\beta_{\text{var}}} = \beta_{\text{app}}$. The modeling of a probability of a source having a Doppler factor $\delta = \beta_{\text{app}}$ from the flux density-limited sample (Cohen et al. 2007) shows that this probability is peaked around unity for a large sample. Another assumption used in this method is that the pattern speed is approximately equal to the flow speed, and the results of modeling by Cohen et al. (2007) support it. The second way to estimate the jet Doppler factor is based on the assumption that the characteristic time of variability of a bright knot in a jet gives us information about

the light-travel time across the knot of the observed angular size. This allows to calculate the variability Doppler factor Jorstad et al. (2005). It has been shown by Jorstad et al. (2005) for the set of 15 sources that $\delta_{\beta_{\text{app}}}$ and δ_{var} correlate with each other, following approximately the linear dependence $\delta_{\beta_{\text{app}}} \approx 0.72 \delta_{\text{var}}$. This supports a possibility of using $\delta_{\beta_{\text{var}}}$ as an estimate for the Doppler factor of each individual source. The third method used by Hovatta et al. (2009) is based on comparison of the variability brightness temperature to the equipartition brightness temperature.

For the two sources with extreme brightness temperature the Doppler factor can be estimated by the first two methods. We do not use the results by Hovatta et al. (2009), since these have been obtained using the equipartition assumption. For 3C 273 source $\delta_{\beta_{\text{app}}} = 14.86$ (Lister et al. 2013) and $\delta_{\text{var}} = 12.6$ (Jorstad et al. 2005). For BL Lac $\delta_{\beta_{\text{app}}} = 9.95$ (Lister et al. 2013) and $\delta_{\text{var}} = 8.1$ (Jorstad et al. 2005). Here we have chosen the maximal value for δ_{var} from the set of different values for different knots. Both methods provide the estimates for the Doppler factors which are in good agreement with each other. For our purposes we use the estimate $\delta_{\beta_{\text{app}}}$, as for the sources under consideration $\delta_{\beta_{\text{app}}} > \delta_{\text{var}}$, thus providing the upper limit for the values of B and Σ and the lower limit for n_{rad} — the closest to the equipartition values limits.

The expression for the observation angle can be found using the Doppler factor definition and equation (38):

$$\varphi = \text{atan} \left(\frac{2\beta_{\text{app}}}{2\beta_{\text{app}}^2 - 1} \right). \quad (39)$$

We also use the observations of apparent half-opening angle by Pushkarev et al. (2009). Knowing the observation angle φ and apparent half-opening angle χ_{app} one can obtain the half-opening angle

$$\chi \approx \chi_{\text{app}} \sin \varphi / 2. \quad (40)$$

The luminosity distance D_L obtained according to the Λ dark matter cosmological model with $H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$ (Komatsu 2009).

BL Lac parameters. For this object we use the brightness temperature measurements by Gómez et al. (2016). We choose the measurement at $\nu_{\text{obs}} = 15 \text{ GHz}$, as this frequency is closest to the frequencies used to estimate a core-shift for this object. The lower estimate for the observed brightness temperature is $7.9 \cdot 10^{12} \text{ K}$. We also employ the following observable parameters needed to obtain the physical properties at the core of BL Lac. They are: $z = 0.069$, $\beta_{\text{app}} = 9.95$ (Lister et al. 2013), $\chi_{\text{app}} = 26.2^\circ$ (Pushkarev et al. 2009), and $\Phi = 0.55 \text{ mas} \cdot \text{GHz}$ (Pushkarev et al. 2012). From these we find $D_L = 0.31 \text{ Gpc}$, $\varphi = 0.1$, $\chi = 0.02$, and $\Gamma \approx 20$. Substituting these parameters into (16), (32) and (37), we obtain: $B = 3.3 \cdot 10^{-2} \text{ G}$, $n_{\text{rad}} = 3.4 \cdot 10^7 \text{ cm}^{-3}$, $\Sigma = 1.3 \cdot 10^{-5}$.

3C273 parameters. For this object we take the measurements of the brightness temperature by Kovalev et al. (2016) at $\nu_{\text{obs}} = 4.8 \text{ GHz}$. For this object the observed brightness temperature is $T_{\text{b, obs}} = 13 \cdot 10^{12} \text{ K}$. We employ the following observable parameters for the 3C273: $z = 0.158$, $\beta_{\text{app}} = 14.86$ (Lister et al. 2013), $\chi_{\text{app}} = 10.0^\circ$ (Pushkarev et al. 2009), and $\Phi = 0.34 \text{ mas} \cdot \text{GHz}$ (Pushkarev et al. 2012). From these we calculate $D_L = 0.75$, $\varphi = 0.067$, $\chi = 0.006$, and $\Gamma \approx 30$. For these parameters we

obtain the following physical parameters: $B = 8.1 \cdot 10^{-3}$ G, $n_{\text{rad}} = 1.4 \cdot 10^7 \text{ cm}^{-3}$, $\Sigma = 2.9 \cdot 10^{-6}$.

The magnitudes of magnetic field and particle number density in radiation region, obtained basing on the brightness temperature measurements, differ significantly from the jet parameters, obtained by Lobanov (1998); O’Sullivan & Gabuzda (2009); Hirotani (2005) basing on the equipartition assumption. From the equations (16), (32) and (37) we can check the would-be observed brightness temperature for the system in equipartition and, consequently, the equipartition magnetic field B_{eq} and radiating particle number density $n_{\text{rad,eq}}$ for our sources. Setting for each source $\Sigma = 1$, which corresponds to the equipartition regime, we obtain for BL Lac $T_{\text{b,eq}} = 1.9 \cdot 10^{12}$ K, and for 3C 273 $T_{\text{b,eq}} = 6.8 \cdot 10^{11}$ K. The equipartition magnetic field and radiating particles number density in the observed core are for BL Lac are $B_{\text{n,eq}} = 0.56$ G and $n_{\text{rad,eq}} = 1.2 \cdot 10^5 \text{ cm}^{-3}$, and for 3C 273 are $B_{\text{n,eq}} = 3$ G and $n_{\text{rad,eq}} = 26 \text{ cm}^{-3}$. The extreme values of the physical parameters of the radiating region are in accordance with the conclusions by Readhead (1994), who had found that even the brightness temperatures at Compton catastrophe limit would need an extreme departures from the equipartition. Here we want to mention that the 3C 273 source demonstrates also the extreme magnitude of Michel’s magnetization, even calculated basing on the equipartition assumptions (Nokhrina et al. 2015).

We have obtained the radiating particles magnetization for the two sources with extreme brightness temperatures to be of the order of 10^{-5} . The obtained radiating magnetization allows us to estimate the total outflow magnetization σ_{tot} . Indeed, the total magnetization is defined as a function of a bulk flow magnetization $\sigma \approx 1$ for MHD outflows and radiating magnetization $\Sigma \ll 1$:

$$\sigma_{\text{tot}} = \frac{B^2}{4\pi mc^2 n \Gamma + 4\pi mc^2 n_{\text{rad}} \ln \frac{\gamma_{\text{max}}}{\gamma_{\text{min}}}} = \frac{1}{1/\sigma + 1/\Sigma}. \quad (41)$$

Thus, we conclude that the radiating plasma must be highly relativistic so as to dominate the particle energy flux, at least in the radiation domain, so that the total outflow magnetization

$$\sigma_{\text{tot}} \approx \Sigma \ll 1. \quad (42)$$

The non-equipartition physical parameters in the core have extreme values. Indeed, let us estimate the maximum particle number density in a jet provided that the total jet power is in particles kinetic energy. Thus,

$$\Gamma mc^3 \int_0^{R_j} n^{\text{lab}}(r_{\perp}) 2\pi r_{\perp} dr_{\perp} \leq P_{\text{jet}}. \quad (43)$$

For the uniform transversal number density distribution we get

$$\left(\frac{n^{\text{lab}}}{\text{cm}^{-3}} \right) \leq 10^4 \left(\frac{P_{\text{jet}}}{10^{45} \text{ erg/s}} \right) \left(\frac{R_{\text{jet}}}{0.1 \text{ pc}} \right)^2. \quad (44)$$

Although the total jet power is not always known, the estimate based on correlation between the total jet power and radio power (Cavagnolo et al. 2010) may be applied. This provides for both sources the values of $P_{\text{BLLac}} \approx 1.2 \cdot 10^{44}$ erg/s and $P_{\text{3C273}} \approx 3.5 \cdot 10^{45}$ erg/s (Nokhrina et al. 2015). For the inequality (44) to hold for the obtained values of n_{rad} , the jet radius R_{jet} has to be approximately 20 pc and

2 pc respectively. These values exceed the measured jet radius for M87 (Mertens et al. 2016) of the order of 0.1 pc. This may mean that we underestimate the magnetic field amplitude, or that the physical conditions in the radiating domain are very different from the conditions over the larger jet domain, so that the Blandford–Königl model is not applicable for the radiating core. Below we will address the first issue of probable underestimation of the magnetic field magnitude.

3 THE SIMPLEST NON-UNIFORM MODEL

We see that the standard approach of Blandford–Königl model applied for the observed extreme brightness temperatures gives the small magnetic field and unphysically high particle number density. However, as it has been pointed out by Marscher (1977), non-uniform models with transversal structure provide the strong dependance of physical parameters of a flow on observables. In this section we will relax the assumption of a uniform distribution of a magnetic field across the radiating domain. Here we will employ the MHD model for the transversal jet structure in the radiating domain in order to calculate the spectral flux and thus obtain the expression for the magnetic field as a function of an observed brightness temperature. We plan to reconsider the effect of non-uniform distribution of physical parameters on core-shift effect in the future paper.

3.1 Model with the uniform velocity across the jet

We assume the radiation site being the part of a continuous cylindrical jet with the bulk Lorentz-factor Γ with plasma excited by some process so it has a power-law energy distribution (6) in the jet frame (which in our model is a pattern frame also). We assume a radiating region of a jet being uniform along the jet axis, but having a transversal structure: magnetic field $B(r_{\perp})$ and particle number density $n(r_{\perp})$ are functions of the radial distance from the jet axis r_{\perp} . These we will specify below in the text.

Modeling the transversal jet structure needs solving the MHD equations — the Grad-Shafranov equation together with the Bernoulli equation (see e.g. review by Beskin (2010)). In general, these cannot be solved analytically, although in some special cases (self-similarity or special geometry) the solution may be obtained. The numerical MHD simulations provide a powerful instrument in constructing the jet internal structure models. In this work we will use the obtained earlier analytical and numerical results as a simplest model for the relativistic jet transverse structure. The particle number density and toroidal magnetic field dependance on the distance from the jet axis we will model by two domains. The first one is a jet central core, which we define as a central part of jet with uniform n and B_{φ} distributions (Komissarov et al. 2007; Lyubarsky 2009; Nokhrina et al. 2015; Tchekhovskoy & Bromberg 2016). The size of a central core R_c is of the order of a few light cylinder radii. In particular, the numerical modeling by Tchekhovskoy & Bromberg (2016) gives for $R_c \approx R_L$, and semi-analytical modeling by (Komissarov et al. 2007; Beskin & Nokhrina 2009; Nokhrina et al. 2015) gives $R_c \approx 5R_L$. As these results are very close, we will use

for simplicity $R_c = R_L$. Further, the same modeling allow us to approximate the particle number density and the poloidal and toroidal magnetic fields in a jet in the second domain by the power laws (Nokhrina et al. 2015; Tchekhovskoy & Bromberg 2016). So, we will use the following functions as an approximation for $B(r_\perp)$ and $n(r_\perp)$:

$$n_{\text{rad}} = n_0 \begin{cases} 1, & r_\perp \leq R_L, \\ (R_L/r_\perp)^2, & R_L < r_\perp \leq R_j, \end{cases} \quad (45)$$

$$B_P = B_0 \begin{cases} 1, & r_\perp \leq R_L, \\ (R_L/r_\perp)^2, & R_L < r_\perp \leq R_j, \end{cases} \quad (46)$$

$$B_\varphi = B_0 \begin{cases} r_\perp/R_L, & r_\perp \leq R_L, \\ R_L/r_\perp, & R_L < r_\perp \leq R_j, \end{cases} \quad (47)$$

For the jet radiating region having the flat Lorentz factor Γ distribution across a jet, the poloidal magnetic field does not change with transformation from the jet into observer's frame, and the toroidal magnetic field transforms as

$$B'_\varphi = B_\varphi/\Gamma. \quad (48)$$

Within this model the poloidal magnetic field dominates the toroidal for $r_\perp < \Gamma R_L$, and we have the following scalings for the particle number density and a magnetic field in a fluid frame:

$$B' = B_0 f_B(r_\perp) = B_0 \begin{cases} 1, & r_\perp \leq R_L, \\ (R_L/r_\perp)^2, & R_L < r_\perp \leq \Gamma R_L, \\ R_L/r_\perp \Gamma, & \Gamma R_L < r_\perp \leq R_j, \end{cases} \quad (49)$$

$$n'_{\text{rad}} = \frac{n_0}{\Gamma} f_n(r_\perp) = \frac{n_0}{\Gamma} \begin{cases} 1, & r_\perp \leq R_L, \\ (R_L/r_\perp)^2, & R_L < r_\perp \leq R_j. \end{cases} \quad (50)$$

Here we also assumed that the ratio of radiating particles to all the particles in a jet is constant across the jet.

The photon emission rate (10) and effective absorption coefficient (11) has been obtained by (Ginzburg & Syrovatskii 1964; Blumenthal & Gould 1970) for the randomly oriented magnetic field. The direction of a magnetic field in the derivation in (Blumenthal & Gould 1970) sets the possible distribution of a pitch angle α of radiating particles. In particular, for randomly oriented field, the pitch angle has a flat distribution, which gives after averaging over α the appropriate factor in function $a(p)$ in (10). However, we can use the expressions (10) and (11) even for the ordered magnetic field, but randomly oriented orbits of the radiating particles, provided the pitch angle α also has a flat distribution.

To obtain the numerical values, we need estimates for R_L and R_j . Further on we use the following dimension parameters for a central engine and an outflow angular velocity: the gravitational radius for a black hole with $M_{\text{BH}} = 10^9 M_\odot$ is $r_g = 10^{-4}$ pc. We also use the result obtained by Zamaninasab et al. (2014) for the light cylinder radius, which can be rewritten as:

$$\frac{\Omega_F r_g}{c} = \frac{2\pi\eta}{50}, \quad (51)$$

where $W_{\text{tot}} = \eta \dot{M} c^2$. Setting $\eta = 1$ we get $R_L \approx 10 r_g$, the result, that we will use. As to jet radius R_j , the observations of M87 provide the value $R_j \approx 0.1$ pc (Mertens et al. 2016), so we set $R_j = 10^2 R_L$.

3.2 Limiting parameters

In the Section 2 we have obtained, that the upper limit for the particle number density in the model with the uniform particle number density distribution is approximately 10^4 cm^{-3} , assuming the jet parameters of the order of $P_{\text{jet}} \approx 10^{45} \text{ erg/s}$ and $R_{\text{jet}} \approx 0.1$ pc. For the non-uniform radial distribution (45) the upper limit on the particle number density amplitude n_0^{lab} is $n_0^{\text{lab}} \leq 10^7$, with $n^{\text{lab}}(R_j)$ being of the order of 10^3 cm^{-3} .

The same bounding limits can be obtained for the toroidal magnetic field — the field that in MHD models defines the Poynting flux transported by a jet:

$$\frac{c}{4\pi} \int_0^{R_j} B_\varphi^2(r_\perp) 2\pi r_\perp dr_\perp \leq P_{\text{jet}}. \quad (52)$$

For the uniform model $B_\varphi \leq 1$ G. For the toroidal magnetic field defined by (47) we obtain for the field amplitude $B_0 \leq 40$ G for the same jet parameters.

3.3 Optical depth for small viewing angles

Let us determine the optical depth

$$\tau = \int_0^{s'_0} \kappa'_\nu ds' \quad (53)$$

of the radiating domain depending on n_0 , B_0 and ν_{obs} for the jets directed almost at the observer — the result applicable for the BL Lac and quasar type sources. Since the optical depth is a Lorentz invariant, we will calculate it in the fluid frame. However, we express it as a function of amplitudes of particle number density n_0 , magnetic field B_0 in the nucleus frame, and frequency ν_{obs} in the observer frame, using the transformations from the jet frame into nucleus or observer frame (13) and (49)–(50). For small viewing angles $\varphi \ll 1$ we simplify the integration by taking $ds' \approx dz'$, so that

$$\begin{aligned} \tau(z, r_\perp) &= 0.28 \frac{1}{f(p)} \left(\frac{\delta}{1+z} \right)^3 \left(\frac{n_0}{\text{cm}^{-3}} \right) \left(\frac{B_0}{\text{G}} \right)^2 \times \\ &\times \left(\frac{\nu_{\text{obs}}}{\text{GHz}} \right)^{-3} \left(\frac{z}{R_L} \right) f_n(r_\perp) f_B^2(r_\perp). \end{aligned} \quad (54)$$

The expression for an optical depth τ (54) can be rewritten through dimensionless τ_0 which depends only on the intrinsic radiating domain parameters and the Doppler factor

$$\tau_0 = 0.28 \frac{1}{f(p)} \left(\frac{\delta}{1+z} \right)^3 \left(\frac{n_0}{\text{cm}^{-3}} \right) \left(\frac{B_0}{\text{G}} \right)^2 \left(\frac{\nu_{\text{obs}}}{\text{GHz}} \right)^{-3}, \quad (55)$$

and the dimensionless ‘position’ factor, so

$$\tau = \tau_0 \frac{z}{R_L} f_n(r_\perp) f_B^2(r_\perp). \quad (56)$$

Since the jet physical parameters change significantly across the jet cross-section, the optical depth of the different domains may be greater or smaller than the unity. For example, let us describe the position of a surface with an optical depth equal to unity along the jet as a function of

radial distance from the jet axis r_\perp . Let us take the observed frequency $\nu_{\text{obs}} = 10$ GHz characteristic for the radio interferometric observations, and $\delta \approx 10$, $\Gamma \approx 10$. For all reasonable parameters of a jet n_0 and B_0 , the surface of $\tau = 1$ is situated at geometrical depth z being only a small fraction of a parsec in the central part of a jet. For greater r_\perp the geometrical depth of the surface $\tau = 1$ grows extremely fast towards the jet edges. The result depends strongly on physical parameters in a jet. For the limiting parameters (the maximal optical thickness), if we assume $n_0 = 10^7 \text{ cm}^{-3}$ and $B_0 = 40 \text{ G}$, the surface $\tau = 1$ for the whole jet cross-section remains optically thick for the radiating domain depth z greater than 10^{-3} pc .

However, for the less extreme parameters the situation is quite different. If we take $n_0 \approx 10^3 \text{ cm}^{-3}$ and $B_0 \approx 1 \text{ G}$ — the equipartition parameters for the uniform model obtained by Lobanov (1998), — the position of the surface with $\tau = 1$ must be of the order of a few parsec at $r_\perp = \Gamma R_L$, which means, that the radiating domain is optically thick in the central jet part and optically thin at the outer jet domain.

3.4 Non-uniform jet velocity

There are some observational indications of a non-flat transversal Lorentz factor structure: the limb-brightening (see e.g. Giroletti et al. (2008)) and M87 observed velocity transverse profile (Mertens et al. 2016). In the latter work there have been detected super-luminal velocities in the limbs as well as in the central stream. The numerical (Tchekhovskoy et al. 2008, 2009) and analytical (Beskin & Nokhrina 2006; Lyubarsky 2009) modeling show that the bulk flow Lorentz factor is not constant in the transversal jet direction. In particular, the following transverse Lorentz factor structure has been predicted by the MHD modeling:

$$\Gamma(r_\perp) = \gamma(r_\perp)\sigma_M = \begin{cases} \gamma_{\text{in}} \approx 1, & r_\perp \leq R_L, \\ r_\perp/R_L, & R_L < r_\perp \leq \sigma_M R_L, \\ \sigma_M, & r_\perp > \sigma_M R_L. \end{cases} \quad (57)$$

Here σ_M is Michel's magnetization parameter — the ratio of Poynting flux to particles kinetic energy flux at the base of an outflow. It bounds the maximum Lorentz-factor as $\Gamma < \sigma_M$. We are using the dependences (45)–(47) for a particle number density and a magnetic field.

The drift bulk velocity of plasma has both a toroidal $v_{\text{dr},\varphi} = v_{\text{dr}} B_P / B_\varphi$ and a poloidal $v_{\text{dr},P} = v_{\text{dr}} B_\varphi / B_P$ components. However, as outside the light cylinder $R_L = \Omega_F / c$ the toroidal magnetic field is much greater than the poloidal, we will neglect by the latter. Thus, the poloidal magnetic field does not change with transformation from the jet into observer's frame for $r_\perp > R_L$, and the toroidal magnetic field transforms as

$$B'_\varphi = B_\varphi / \Gamma(r_\perp). \quad (58)$$

Inside the light cylinder we have the opposite: we transform the poloidal magnetic field, and the toroidal field is unchanged. So, under these assumptions in the fluid frame the toroidal magnetic field dominates the poloidal one at

$r_\perp > R_L$, and we have the following magnetic field and particle number density transversal profiles in the jet frame:

$$B' = B_0 \begin{cases} 1, & r_\perp \leq R_L, \\ (R_L/r_\perp)^2, & R_L < r_\perp \leq \sigma_M R_L, \\ R_L/r_\perp \sigma_M, & \sigma_M R_L < r_\perp \leq R_j, \end{cases} \quad (59)$$

$$n'_{\text{rad}} = n_0 \begin{cases} 1, & r_\perp \leq R_L, \\ (R_L/r_\perp)^3, & R_L < r_\perp \leq \sigma_M R_L, \\ (R_L/r_\perp)^2 / \sigma_M, & \sigma_M R_L < r_\perp \leq R_j. \end{cases} \quad (60)$$

The flow Doppler factor depends on the distance from the axis as well. We introduce the Doppler factor for the fastest part of a flow $\delta_0 = 1/\sigma_M(1 - \beta(r_\perp) \cos \theta)$. As the flow is relativistic, we neglect by the change in β across the flow, and use

$$\delta(r_\perp) = \frac{\delta_0}{\gamma(r_\perp)}. \quad (61)$$

Due to this, the observed spectral flux will be much less homogeneous than it is suggested by mere change in B' and n' in comparison with the model with the uniform jet velocity.

The optical thickness as a function of z and r_\perp is now given by

$$\begin{aligned} \tau(z, r_\perp) &= \frac{0.28}{f(p)} \left(\frac{\delta_0}{1+z} \right)^3 \left(\frac{n_0}{\text{cm}^{-3}} \right) \left(\frac{B_0}{\text{G}} \right)^2 \times \\ &\times \left(\frac{\nu_{\text{obs}}}{\text{GHz}} \right)^{-3} \left(\frac{z}{R_L} \right) \frac{f_n(r_\perp) f_B^2(r_\perp)}{\gamma^3(r_\perp)} = \\ &= \tau_{0,2} \frac{z}{R_L} \frac{f_n(r_\perp) f_B^2(r_\perp)}{\gamma^3(r_\perp)}. \end{aligned} \quad (62)$$

This equation coincides with (54) except for additional factor $\gamma^3(r_\perp)$. This means that the optical depth in the central jet part is the same as in a limit of a uniform Lorentz factor. However, the position of a surface $\tau = 1$ grows much more rapidly for $r_\perp > R_L$, and for reasonable depth L of radiating domain it becomes optically thin. The equation for surface $\tau = 1$ is given by

$$\frac{z(r_\perp)}{R_L} = \frac{1}{\tau_{0,2}} \begin{cases} \sigma_M^{-3}, & r_\perp \leq R_L, \\ (r_\perp/R_L)^{10} \sigma_M^{-3}, & R_L < r_\perp \leq \sigma_M R_L, \\ (r_\perp/R_L)^4 \sigma_M^3, & \sigma_M R_L < r_\perp \leq R_j. \end{cases} \quad (63)$$

Although the radiating domain optical thickness declines more rapidly than in the case of a flat velocity distribution, for upper limits for n_0 and B_0 the outer part of an outflow stays optically thick for $L < 2 \cdot 10^{-2} \text{ pc}$.

3.5 Observed flux

Now we will calculate the observed spectral flux of a model radiating domain with the non-uniform distribution of a particle number density and a magnetic field for small viewing angles. We first determine the spectral flux in the fluid frame, where an emissivity and effective absorption are readily calculated (Ginzburg & Syrovatskii 1964;

Blumenthal & Gould 1970), and then transform it into the observer frame (Rybicki & Lightman 1979).

A spectral flux in the jet frame is defined by

$$S'_{\nu'} = \frac{1}{d^2} \int_{\Omega'} j'_{\nu'}(\nu') dV' e^{-\int \kappa'_{\nu'}(\nu') ds'}, \quad (64)$$

where Ω' is a radiating domain. Using (10) and (11) for the synchrotron radiation, and having $j'_{\nu'}(\nu') = \hbar \nu' \rho'_{\nu'}(\nu')$, we obtain for the flux in a jet frame written through the observed frequency ν_{obs} and a particle number density n and a magnetic field B in the nucleus frame the following expression:

$$S'_{\nu}(\nu, n_0, B_0) = 0.16 \frac{\hbar \nu}{d^2} \frac{\nu}{r_0 c} \left(\frac{\nu_{B_0}}{\nu} \right)^{-1/2} \left(\frac{1+z}{\delta} \right)^{5/2} I, \quad (65)$$

where the integral I has dimension cm^2 and is defined by

$$I = \int_0^{R_j} \frac{1}{\sqrt{f_B(r_{\perp})}} r_{\perp} dr_{\perp} \left[1 - e^{-\tau_0 \frac{r_{\perp}}{R_L} f_n(r_{\perp}) f_B^2(r_{\perp})} \right]. \quad (66)$$

If the whole jet cross-section is optically thick, the integral can be easily calculated. In the inner domain $r_{\perp} \in [0, \Gamma R_L]$ it is equal

$$I_{\text{in}} = R_L^2 \left(\frac{1}{6} + \frac{\Gamma^3}{3} \right), \quad (67)$$

and in the outer domain $r_{\perp} \in (\Gamma R_L, R_j]$

$$I_{\text{out}} = \frac{2}{5} R_L^2 \left(\sqrt{\Gamma} \left(\frac{R_j}{R_L} \right)^{5/2} - \Gamma^3 \right), \quad (68)$$

and

$$I \approx \frac{2}{5} \sqrt{\Gamma} \sqrt{\frac{R_j}{R_L}} R_j^2, \quad (69)$$

the outer radiating domain providing the major part of the total flux.

In order to link the spectral flux in the jet frame with the observed brightness temperature, we use the Lorentz invariance of S_{ν}/ν^3 (Rybicki & Lightman 1979). Substituting (65) and (69) into (8), one obtains:

$$\left(\frac{B_0}{\text{G}} \right) = 6.4 \cdot 10^{-4} \Gamma \frac{R_j}{R_L} \frac{\delta}{1+z} \left(\frac{\nu_{\text{obs}}}{\text{GHz}} \right) \left(\frac{T_{\text{b,obs}}}{10^{12} \text{ K}} \right)^{-2}. \quad (70)$$

Compare this result with the uniform model (16). The non-uniform model of optically thick outflow gives for a magnetic field the amplitude of the uniform model multiplied by a “geometrical” factor R_j/R_L , which raise the value by two orders. We see that the uniform model underestimates even the average value of a magnetic field in comparison with the non-uniform model. Indeed, B_0 is greater than the uniform magnetic field everywhere across the jet, and for $R_j/R_L = 10^2$ both fields become comparable only at the jet boundary.

The magnetic field estimated in the frame of a model with non-uniform jet velocity distribution obeys the same expression (70), since the outer domain $r_{\perp} > \sigma_M R_L$ contributes most in a spectral flux, and the velocity profile in this domain is the same for two models.

For the two sources with the measured extreme brightness temperatures, the magnetic field estimated by the non-uniform model is:

$$B_{\text{non-uni}}^{\text{BL Lac}} = 3 \text{ G}, \quad B_{\text{non-uni}}^{3\text{C}273} = 0.7 \text{ G}. \quad (71)$$

4 ASTROPHYSICAL APPLICATIONS AND DISCUSSIONS

In the frame of Blandford–Königl model we have rederived the expressions for a magnetic field and a particle number density used by (Zdziarski et al. 2015) as a tool to estimate these physical parameters of radiation domain in a jet independently of the equipartition assumption. However, contrary to their work, we expressed the parameters through the brightness temperature. As the values for $T_{\text{b,obs}}$ obtained with high resolution exceed by two orders the equipartition temperature derived by Readhead (1994), the jet parameters differs from the ones corresponding to equipartition, the measure of equipartition being of the order of $10^{-6} \div 10^{-5}$. In particular, the magnetic field in radiating domain has an order of 10^{-3} , which according to (7) provides the magnetic field at the gravitational radius B_g of the order of a few G. The expression for n gives the unphysically high amount of particles of the order of 10^7 cm^{-3} , since such an amount would carry energy exceeding the total jet power. However, the two sources regarded in this work have core-shifts smaller than typical errors estimated in Pushkarev et al. (2012) of 0.05 mas. Thus, the results for Σ and n may be subject to big errors.

We have obtained the expression for the magnetic field amplitude B_0 that can be estimated by measurement of a brightness temperature. The expression is applicable for blazars, since it uses the head-on model of radiation transfer for the non-uniform cylindrical optically thick radiation domain with profiles for particle number density and magnetic field distribution based on MHD modeling. The field amplitude characterizes the radiating core region only and may differ from the other domains along the jet. The expression for B_0 differs from the expression for the homogeneous model of the radiating domain by the factor R_j/R_L , which gives two orders of magnitude.

In the frame of MHD models the amplitude B_0 characterizes both poloidal and toroidal magnetic field. Thus, this amplitude provides us with an instrument of checking the electrodynamic model of the black hole energy extraction. Indeed, if we assume the unipolar inductor model for the AGN, the total jet power is given by (Beskin 2010)

$$P_{\text{tot}} = \left(\frac{\Omega r_g}{c} \right)^2 B_g^2 r_g^2 c. \quad (72)$$

As we can estimate B_0 as being of the order of 1 G, using (46), we can correlate the magnetic field amplitude with the total flux crossing the gravitational radius, and thus obtain the poloidal magnetic field at the base of an outflow B_g . Indeed, on the one hand,

$$\Psi_{\text{tot}} = \pi B_g r_g^2, \quad (73)$$

and on the other hand

$$\Psi_{\text{tot}} \approx 2\pi B_0 R_L^2 \ln \frac{R_j}{R_L}. \quad (74)$$

From these equalities we have

$$B_g = 2B_0 \ln \frac{R_j}{R_L} \left(\frac{R_L}{r_g} \right)^2, \quad (75)$$

which gives $B_g \approx 10^3$. It is about an order smaller than the Eddington magnetic field (see, e.g., Beskin (2010))

$$B_{\text{Edd}} = 10^4 \left(\frac{M_{\text{BH}}}{10^9 M_{\odot}} \right)^{-1/2} \text{ G.} \quad (76)$$

Such a magnitude for B_g provides within the electrodynamic model for the total jet power an estimate $P_{\text{tot}} \approx 3 \cdot 10^{43} \text{ erg/s}$.

The above expression (75) for a poloidal magnetic field magnitude is consistent with the maximum possible amount of magnetic flux achieved by magnetically arrested disks. Indeed, for $M_{\text{BH}} = 10^9 M_{\odot}$ and accretion rate 10% of Eddington luminosity (Hawley et al. 2015), $\Phi_{\text{MAD}} = 3 \cdot 10^{33} \text{ G cm}^2$, and for B_g given by (71) and (75) the magnetic flux is of the order of 10^{33} G cm^2 .

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REFERENCES

- Abdo A.A. et al., 2011, *ApJ*, 736, 131
Beskin V.S., 2007, *Soviet Physics Uspekhi*, 40, 659
Beskin V.S., 2010, *Physics-Uspekhi*, 53, 1199
Beskin V.S., Nokhrina E.E., 2006, *MNRAS*, 367, 375
Beskin V.S., Nokhrina E.E., 2009, *MNRAS*, 397, 1486
Blandford R.D., Königl A., 1979, *ApJ*, 232, 34
Blandford R.D., Ostriker J.P., 1978, *ApJ*, 221, L29
Blumenthal G.B. & Gould R.J., 1970, *Reviews of Modern Physics*, 42, 237
Cavagnolo K.W., McNamara B.R., Nulsen P.E.J., Carilli C.L., Jones C., Birzan L., 2010, *ApJ*, 720, 1066
Cohen M.H., Lister M.L., Homan D.C., Kadler M., Kellermann K.I., Kovalev Y.Y., Vermeulen R.C., 2007, *ApJ*, 658, 232
Ginzburg V.L. & Syrovatskii S.I., 1964, *Ann. Rev. Astr. Ap.*, 3, 297
Giroletti M., Giovannini G., Cotton W.D., Taylor G.B., Pérez-Torres M.A., Chiaberge M., Edwards P.G., 2008, *A&A*, 488, 905
Gould R.J., 1979, *A&A*, 76, 306
Gómez J.L., Lobanov A.P., Bruni G., Kovalev Y.Y. et al., 2016, *ApJ*, 817, 96
Hawley J.F., Fendt C., Hardcastle M., Nokhrina E., Tchekhovskoy A. 2015, *Space Sci Rev*, 191, 441
Hirotani K., 2005, *ApJ*, 619, 73
Hovatta T., Valtaoja E., Tornikoski M., Lähteenmäki A., 2009, *A&A*, 494, 527
Jorstad S.G. et al., 2005, *AJ*, 130, 1418
Lind K.R., Blandford R.D., 1985, *ApJ*, 295, 358
Lister M.L. et al., 2009, *AJ*, 137, 3718
Lister M.L. et al., 2013, *AJ*, 146, 120
Lobanov A.P., 1998, *A&A*, 330, 79
Lobanov A.P., 2015, *A&A*, 574, A84
Lyubarsky Yu., 2009, *ApJ*, 698, 1570
Kellermann K.I., Pauliny-Toth I.I.K., 1969, *ApJ*, 155, L71
Kirk J.G., Melrose D.B., Priest E.R., 1994, *Plasma Astrophysics — Saas-Fee Advanced Course 24*, Springer-Verlag
Komatsu E. et al., 2009, *ApJS*, 180, 330
Komissarov S.S., Barkov M.V., Vlahakis N., Königl A., 2007, *MNRAS*, 380, 51
Kovalev Y.Y., Kardashev N.S., Kellermann K.I. et al., 2016, *ApJ Letters*, 820, L9
Marscher A.P., 1977, *ApJ*, 216, 244
Mertens F., Lobanov A.P., Walker R.C., Hardee P.E., 2016, *A&A*, 595, 54
Michel F.C., 1969, *ApJ*, 158, 727
Narayan R., McKinney J.C., Farmer A.J., 2007, *MNRAS*, 375, 548
Nokhrina E.E., Beskin V.S., Kovalev Y.Y., Zheltoukhov A.A., 2015, *MNRAS*, 447, 2726
O’Sullivan S.P., Gabuzda D.C., 2009, *MNRAS*, 400, 26
Pushkarev A.B., Kovalev Y.Y., Lister M.L., Savolainen T., 2009, *A&A*, 507, L33
Pushkarev A.B., Hovatta T., Kovalev Y.Y. et al. 2011, *A&A*, 545, A113
Readhead A.C.S., 1994, *ApJ*, 426, 51
Rybicki G.B., Lightman A.P., 1979, *Radiative Processes in Astrophysics*, Wiley-Vch
Scheuer P.A., Williams P.J.S., 1968, *ARA&A*, 6, 321
Sironi L., Spitkovsky A., 2011, *ApJ*, 726, 75
Sironi L., Spitkovsky A., Arons J., 2013, *ApJ*, 771, 54
Sironi L., Spitkovsky A., 2014, *ApJ*, 783, L21
Sokolovsky K.V. et al., 2011, *A&A*, 532, A38
Tchekhovskoy A. McKinney J.C., Narayan R., 2008, *MNRAS*, 388, 551
Tchekhovskoy A. McKinney J.C., Narayan R., 2009, *ApJ*, 699, 1789
Tchekhovskoy A. & Bromberg O., 2016, *MNRAS*, 461, 46
Zdziarski A.A., Sikora M., Pjanka P., Tchekhovskoy A., 2015, *MNRAS*, 451, 927
Zamaninasab M., Clausen-Brown E., Savolainen T., Tchekhovskoy A., 2014, *Nature*, 510, 126